Canonical Form, Minterms & Maxterms

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Canonical Forms: Sum of Products with Two Variables Showing Minterms

Minterm	A	B	Result
m_0	0	0	r_0
m_1	0	1	r_1
m_2	1	0	r_2
m_3	1	1	r_3

$$Result(A,B) = r_0\bar{A}\bar{B} + r_1\bar{A}B + r_2A\bar{B} + r_3AB$$

The minterms are:

$m_0 = \bar{A}\bar{B}$ $m_2 = A\bar{B}$

$m_1 = \bar{A}B$ $m_3 = AB$

Because we know the values of r_0 through r_3, those minterms where r_n is equal to 0 are omitted

Canonical Forms: Sum of Products with Three Variables

A	B	C	Result
0	0	0	r_0
0	0	1	r_1
0	1	0	r_2
0	1	1	r_3
1	0	0	r_4
1	0	1	r_5
1	1	0	r_6
1	1	1	r_7

$$Result(A,B,C) = r_0\bar{A}\bar{B}\bar{C} + r_1\bar{A}\bar{B}C + r_2\bar{A}B\bar{C} + r_3\bar{A}BC + r_4A\bar{B}\bar{C} + r_5A\bar{B}C + r_6AB\bar{C} + r_7ABC$$

Canonical Forms: Sum of Products Example of Full Adder Using Sigma Minterm Notation

A	B	$Carry_{in}$	Sum	$Carry_{out}$
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

$Sum(A, B, Cin) = [m_1]\bar{A}\bar{B}C + [m_2]\bar{A}B\bar{C} + [m_4]A\bar{B}\bar{C} + [m_7]ABC = \Sigma$ m(1, 2, 4, 7)

$Carry_{out}(A, B, Cin) = [m_3]\bar{A}BC + [m_5]A\bar{B}C + [m_6]AB\bar{C} + [m_7]ABC = \Sigma$ m(3, 5, 6, 7)

[The square brackets enclose comments]

Canonical Forms: Product of Sums with Two Variables Showing Maxterms

Maxterm	A	B	Result
M_0	0	0	r_0
M_1	0	1	r_1
M_2	1	0	r_2
M_3	1	1	r_3

The maxterm for a row is the OR of each variable – uncomplemented if it is a 0 and complemented if it is a 1.
AND together those maxterms where the Result is a 0.

Thus, a maxterm is the complement of the corresponding minterm; and, because we are selecting those rows where the Result is a 0, we are performing an additional complementation. This is tantamount to applying DeMorgan's Law to the Sum of Products formulation.

The maxterms are:

$M_0 = (A + B)$

$M_1 = (A + \bar{B})$

$M_2 = (\bar{A} + B)$

$M_3 = (\bar{A} + \bar{B})$

Canonical Forms: Conversion of Product of Sums to Sum of Products

Maxterm	A	B	Result
M_0	0	0	r_0
M_1	0	1	r_1
M_2	1	0	r_2
M_3	1	1	r_3

$Result(A,B) = \overline{r_0}(A+B) \cdot \overline{r_1}(A+\bar{B}) \cdot \overline{r_2}(\bar{A}+B) \cdot \overline{r_3}(\bar{A}+\bar{B})$
$= \Pi$ M(where the Result, r_n, is 0)

Convert into Sum of Products form by choosing those rows where the Results are 1 rather than 0,

$Result(A,B)$
$= r_0(\bar{A}\bar{B}) + r_1(\bar{A}B) + r_2(A\bar{B}) + r_3(AB) = \Sigma$ m(where the Result, r_n, is 1)

Canonical Forms: Product of Sums with Three Variables

A	B	C	Result
0	0	0	r_0
0	0	1	r_1
0	1	0	r_2
0	1	1	r_3
1	0	0	r_4
1	0	1	r_5
1	1	0	r_6
1	1	1	r_7

$$Result(A,B,C) = \overline{r_0}(A+B+C) \cdot \overline{r_1}(A+B+\bar{C}) \cdot \overline{r_2}(A+\bar{B}+C) \cdot \overline{r_3}(A+\bar{B}+\bar{C}) \cdot \overline{r_4}(\bar{A}+B+C) \cdot \overline{r_5}(\bar{A}+B+\bar{C}) \cdot \overline{r_6}(\bar{A}+\bar{B}+C) \cdot \overline{r_7}(\bar{A}+\bar{B}+\bar{C})$$

Full Adder Using Sum of Products and Product of Sums

- [The square brackets enclose comments]
- Sum of Products Using Sigma Minterm Notation
  - $Sum(A, B, Cin) = [m_1]\bar{A}\bar{B}C + [m_2]\bar{A}B\bar{C} + [m_4]A\bar{B}\bar{C} + [m_7]ABC =$ Σ m(1, 2, 4, 7)
  - $Carry_{out}(A, B, Cin) = [m_3]\bar{A}BC + [m_5]A\bar{B}C + [m_6]AB\bar{C} + [m_7]ABC =$ Σ m(3, 5, 6, 7)
- Product of Sums Using Pi Maxterm Notation
  - $Sum(A, B, Cin) = [M_0](A + B + C) \cdot [M_3](A + \bar{B} + \bar{C}) \cdot [M_5](\bar{A} + B + \bar{C}) \cdot [M_6](\bar{A} + \bar{B} + C) =$ Π M(0, 3, 5, 6)
  - $Carry_{out}(A, B, Cin) = [M_0](A + B + C) \cdot [M_1](A + B + \bar{C}) \cdot [M_2](A + \bar{B} + C) \cdot [M_4](\bar{A} + B + C) =$ Π M(0, 1, 2, 4)